Valuing Non-Interest-Bearing Liabilities
Growth Rate is Constant

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In this white paper we will place a value on a company's non-interest-bearing liabilities (NIBL). We will assume that NIBL (cash received or expenses deferred) are invested at a given investment yield. The value of NIBL in this case is the present value of after-tax investment income. We will further assume that the growth rate of the NIBL balance is constant. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are currently standing at time zero and are tasked with determining the value of a ABC Company's non-interest-bearing liabilities. Our go-forward model assumptions are...

Description	Value
Notional value at time zero ($)	1,000,000
Annual notional value growth rate (%)	3.50
Annual risk-free investment yield (%)	4.25
Annual risk-adjusted discount rate (%)	9.50
Income tax rate (%)	15.50
Ratio of NIBL to notional value (%)	20.00

Note: Notional value is defined as tangible assets for banks and annualized revenue for non-banks.

Question 1: What is the book value of non-interest-bearing liabilities at time zero?

Question 2: What is the market value of non-interest-bearing liabilities at time zero?

NIBL Investment Income

We will define the variable N_t to be notional value at time t and the variable ω to be the continuous-time notional value growth rate. The equation for notional value at time t is...

$$N_t = N_0 \operatorname{Exp}\left\{\omega\, t\right\} \tag{1}$$

Using Equation (1) above, the equation for the continuous-time notional value growth rate is...

$$\omega = \ln\left(1 + \text{Discrete-time growth rate}\right) \tag{2}$$

Using Equations (1) and (2) above, the equation for the book value of non-interest-bearing liabilities at time t is...

$$NIBL_t = \eta\, N_t = \eta\, N_0 \operatorname{Exp}\left\{\omega\, t\right\} \tag{3}$$

We will define the variable I_t to be annualized after-tax investment income at time t, the variable η to be the ratio of NIBL to notional value, the variable α to be the continuous-time investment coupon rate, and the variable τ to be the income tax rate. Using Equation (1) above, the equation for annualized after-tax income at time t is...

$$I_t = \alpha\,(1-\tau)\,\eta\, N_t = \alpha\,(1-\tau)\,\eta\, N_0 \operatorname{Exp}\left\{\omega\, t\right\} \tag{4}$$

Using Equation (4) above, the equation for the continuous-time investment coupon rate is...

$$\alpha = \ln \left(1 + \text{Discrete-time investment coupon rate} \right) \tag{5}$$

We will define the variable $I_{m,n}$ to be after-tax investment income recognized over the time interval $[m, n]$. Using Equation (4) above, the equation for cumulative after-tax investment income is...

$$I_{m,n} = \int\limits_{m}^{n} I_t \, \delta t = \alpha \, (1 - \tau) \, \eta \, N_0 \int\limits_{m}^{n} \text{Exp} \left\{ \omega \, t \right\} \delta t \tag{6}$$

The solution to Equation (6) above when the notional value growth rate is non-zero is...

$$I_{m,n} = \frac{\alpha \, (1 - \tau)}{\omega} \, \eta \, N_0 \left[\text{Exp} \left\{ \omega \, n \right\} - \text{Exp} \left\{ \omega \, m \right\} \right] \text{ ...given that... } \omega \neq 0 \tag{7}$$

The solution to Equation (6) above when the notional value growth rate is zero is... [1]

$$I_{m,n} = \alpha \, (1 - \tau) \, \eta \, N_0 \left[n - m \right] \tag{8}$$

NIBL Valuation

We will define the variable κ to be the continuous-time, risk-adjusted discount rate. The equation for the discount rate is...

$$\kappa = \ln \left(1 + \text{Annual discount rate} \right) \tag{9}$$

Using Equations (4) and (9) above, the equation for the present value of non-interest-bearing liabilities over the time interval $[0, T]$ is...

$$V_{0,T} = \int\limits_{0}^{T} I_t \, \text{Exp} \left\{ - \kappa \, t \right\} \delta t = \alpha \, (1 - \tau) \, \eta \, N_0 \int\limits_{0}^{T} \text{Exp} \left\{ (\omega - \kappa) \, t \right\} \delta t \text{ ...given that... } \omega < \kappa \tag{10}$$

The solution to Equation (10) above is...

$$\begin{aligned} V_{0,T} &= \frac{\alpha \, (1 - \tau)}{\omega - \kappa} \, \eta \, N_0 \left[\text{Exp} \left\{ (\omega - \kappa) \times T \right\} - \text{Exp} \left\{ (\omega - \kappa) \times 0 \right\} \right] \\ &= \frac{\alpha \, (1 - \tau)}{\kappa - \omega} \, \eta \, N_0 \left[1 - \text{Exp} \left\{ (\omega - \kappa) \, T \right\} \right] \end{aligned} \tag{11}$$

Using Equation (12) above, the solution to Equation (10) as the upper integral bound goes to infinity is...

$$V_{0,\infty} = \frac{\alpha \, (1 - \tau)}{\kappa - \omega} \, \eta \, N_0 \left[1 - \text{Exp} \left\{ (\omega - \kappa) \times \infty \right\} \right] = \frac{\alpha \, (1 - \tau)}{\kappa - \omega} \, \eta \, N_0 \tag{12}$$

Answers To Our Hypothetical Problem

Using our go-forward model assumptions above, our continuous-time rates are...

$$\alpha = \ln \left(1 + 0.0425 \right) = 0.0416 \text{ and } \omega = \ln \left(1 + 0.0350 \right) = 0.0344 \text{ and } \kappa = \ln \left(1 + 0.0950 \right) = 0.0908 \tag{13}$$

Question 1: What is the book value of non-interest-bearing liabilities at time zero?

Using Equation (3) above and the go-forward model assumptions above, the answer to the question is...

$$\text{Book value at time zero} = 0.20 \times 1{,}000{,}000 = 200{,}000 \tag{14}$$

Question 2: What is the market value of non-interest-bearing liabilities at time zero?

Using Equations (12) and (13) above and the go-forward model assumptions above, the answer to the question is...

$$V_{0,\infty} = \frac{0.0416 \times (1 - 0.1550)}{0.0908 - 0.0344} \times 0.20 \times 1{,}000{,}000 = 124{,}652 \tag{15}$$

References

[1] Gary Schurman, *An Application of L'Hopital's Rule - Growth Rate is Zero,* May, 2025